The Baumol - Tobin model of money demand

As a starting point, consider the simple theory of money demand expressed by the exchange equation:

$$PY = VM \tag{1}$$

It states that money demand is proportional to the nominal value of all goods sold on the market. The more goods Y there is to buy, and the higher the prices P of those goods, the more money is needed. The factor of proportionality V is called the velocity of money.

A closer analysis reveales that money demand also depends on the interest rate r.

$$M = M^d(Y, r) \tag{2}$$

Money demand increases with Y and decreases with r.

$$\frac{\partial M^d}{\partial Y}(Y, r) > 0 \tag{3}$$

$$\frac{\partial M^d}{\partial r}(Y, r) < 0 \tag{4}$$

The Baumol - Tobin model of money demand presented here is consistent with the exchange equation, with the additional feature that the velocity of money depends on the interest rate.

Suppose households recieve income PY in the beginning of each period, and spend it evenly during the period. (No savings.) Average wealth is:

$$W = \frac{PY}{2} \tag{5}$$

Households hold wealth in money, which is liquid but earns no interest, and bonds, yielding interest at a rate r.

$$W = M + B \tag{6}$$

Before wealth in bonds can be spent, it has to be changed into money at an transaction fee F per transaction, which is regarded as fixed by the individual household, i.e. independent of the amount transfered.

Suppose the household divides the period into n subperiods, initially placing

$$\frac{PY}{n}$$

in money and the rest in bonds. At the end of each subperiod, bonds are converted to money in $n-1$ transactions of equal size $\frac{PY}{n}$.

Average money holdings over a full period of n subperiods are

$$M = \frac{1}{n}\frac{PY}{2} \tag{7}$$

Average bonds holdings are

$$B = \frac{(n-1)}{n}\frac{PY}{2} \tag{8}$$

Interest earned on bonds is rB. Transaction costs are $(n-1)F$. The net gain is:

$$N = r\frac{(n-1)}{n}\frac{PY}{2} - (n-1)F \tag{9}$$

Maximizing net gain with respect to n requires:

$$\frac{dN}{dn} = r\frac{1}{n^2}\frac{PY}{2} - F = 0 \tag{10}$$

So the optimal choice of n is:

$$n = \sqrt{\frac{rPY}{2F}} \tag{11}$$

The equation (11) for optimal n, inserted into the equation (7) for the money holdings, give money demand as a function of nominal income PY and the ineterest rate r:

$$M = \sqrt{\frac{FPY}{2r}} \tag{12}$$

Thus, money demand decreases with the interest rate r.

Furthermore, if trasaction costs grow with nominal aggregate income in the long run, allthough unaffected by individual households' decisions, $F = fPY$, the money demand equation (12) is transformed into:

$$M = PY\sqrt{\frac{f}{2r}} \tag{13}$$

Equation (13) is the exchange equation (1), with a velocity V increasing with the interest rate:

$$V = \sqrt{\frac{2r}{f}} \tag{14}$$

Figure 1 illustrates equilibrium on the money market, for two different levels of income Y_1 and Y_2, where $Y_1 < Y_2$.

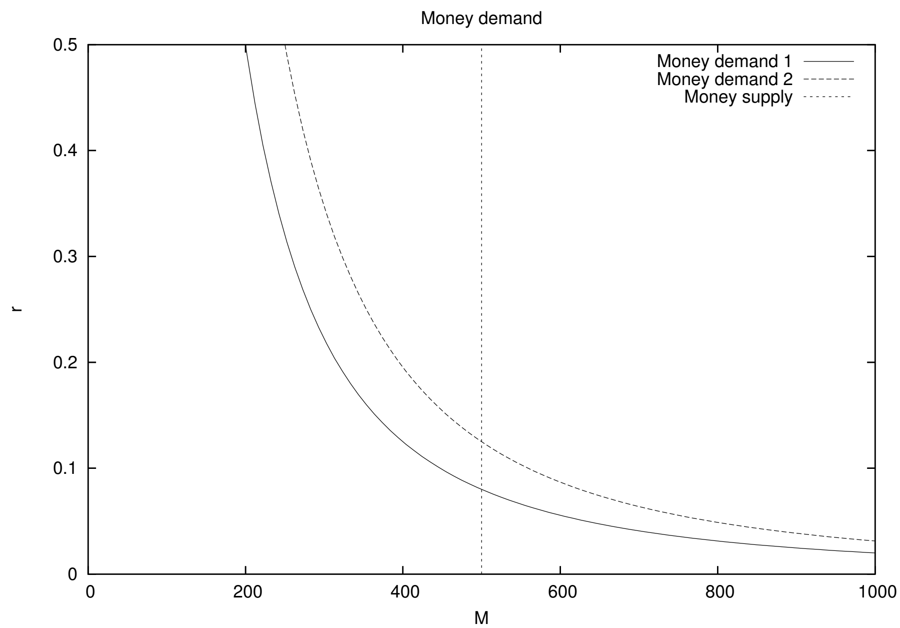

Figure 1: The money market