A simple economy in long run static equilibrium

Introduction

These notes describe a simple economy, maybe even the simplest possible, whith sufficient structure to determine many of the most important macroeconomic variables from microeconomic relationships. In order to bring out the logic structure of the theory, the model has been simplified to the extent that trying to choose its parameters to fit it to real macroeconomic datea, e.g. the national accounts, would be futile. On the other hand, any realistic macroeconomic model with microeconomic foundations would have to include variants of the relationships in this model.

The simplifications are of different kinds. The most severe is the restriction to a closed private economy, excluding the public sector and trade with the rest of the world. This is what renders impossible an adjustment to the national accounts. The restriction to static equilibrium limits it the analysis to the long run, but it easy to extend the model to include growth as well as adjustment dynamics. Labor supply, which is here fixed, should be derived from the households' preferences. A realistic labor market also requilres some imperfection to explain unemployment.

A list of additions required to make the model useful to describe the develpopment of a real economy in the medium and the long run would look something like this:

- a public sector with consumption, taxes, transfers, borrowing ...
- a foreign sector: exports, imports, exchange rates, capital flows ...
- dynamics with growth and adjustments to long run steady state
- an endogenous labor supply
- an imperfect labor market with unemployment (equilibrium and dynamics)
- money and a central bank, inflation
- a Phillips curve
- financial assets

The production function

Output in the long run is determined by the amounts of the production factors available and by the technology with which they are combined. A production function describes the technology by giving production (Y) as an increasing function of the productive factors: capital (K), labor (L) and natural resources (N).

$$Y_t = F(K_t, L_t, N_t)$$

As the National Accounts don't contain data on natural resources, we simplify the production function to just capital and labor.

$$Y_t = F(K_t, L_t)$$

Aggregate production functions typically exhibit constant returns to scale. i.e. if all productive factors increase in the same proportion (s), production also increase in that proportion.

$$sY_t = F(sK_t, sL_t)$$

Example: the Cobb-Douglas production function

$$Y_t = A_t K_t^a L_t^{1-a}$$

It has constant returns to scale and fits aggregate data reasonably well. The exponent a is equal to the capital share of income.

The marginal product of factors

With constant returns to scale, if all factors increase proportionally, production increase in the same proportion. Thus if only one factor increases, while the other reamains constant, the production increases less than proportionally. The derivative of the production function with respect to one of the factors is called the marginal product of that factor. The marginal product measures the increase in production from an additional unit of the factor.

The marginal product of capital (MPK) is defined as:

$$MPK = \frac{\partial F}{\partial K}(K_t, L_t) \approx \frac{\Delta Y}{\Delta K}$$

and the marginal product of labor as:

$$MPL = \frac{\partial F}{\partial L}(K_t, L_t) \approx \frac{\Delta Y}{\Delta L}$$

Since the increase in production from additional units of one single factor is less than proportional, the marginal product is decreasing.

$$\frac{\partial}{\partial K} MPK = \frac{\partial\partial F}{\partial K^2}(K_t, L_t) < 0$$

$$\frac{\partial}{\partial L} MPL = \frac{\partial\partial F}{\partial L^2}(K_t, L_t) < 0$$

The labor market

Labor supply is assumed fixed at $L = \overline{L}$.

The marginal product of labor (MPL) gives the firms' labor demand function. The gain to the firm from hiring an additional worker is his marginal product, while the additional cost is his wage w. As long as gains (MPL) exceeds costs (w), firms increase their labor demand. Since the marginal product decreases with increased labor, eventually the equilibrium point is reached where $MPL = w$. If the wage exceeds the marinal product, firms reduce their labor force, increasing the marginal product of labor.

Wages w are assumed to adjust flexibly to make demand match supply. If labor demand exceeds labor supply, competition among firms for workers drive up the

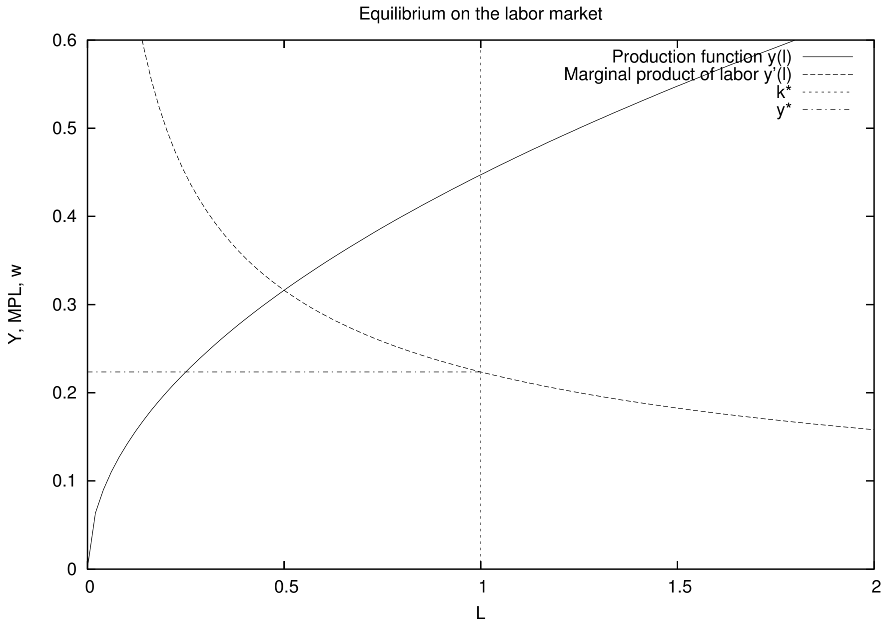

Figure 1: The labor market

wage. Likewise, if supply exceeds demand, competition among workers for jobs drive the wage up.

Figure 1 illustrates equilibrium on the labormarket with constant labor supply, decreasing labor demand, and flexible wages.

Capital formation

Proportional capital depreciation with depreciation rate δ

$$K_{t+1} = (1-\delta)K_t + I_t$$

Equilibrium with constant caipital stock: $K_{t+1} = K_t$

$$I_t = \delta K_t$$

Cost of capital: depreciation + interest

$$c(K_t) = (r+\delta)K_t$$

Gain from capital: the marginal product of capital

Equilibrium: marginal gain = marginal costs

$$MPK = r + \delta$$

The consumption – savings decision

Households divide their income into consumption and savings. To motivate savings, at least two periods must be considered.

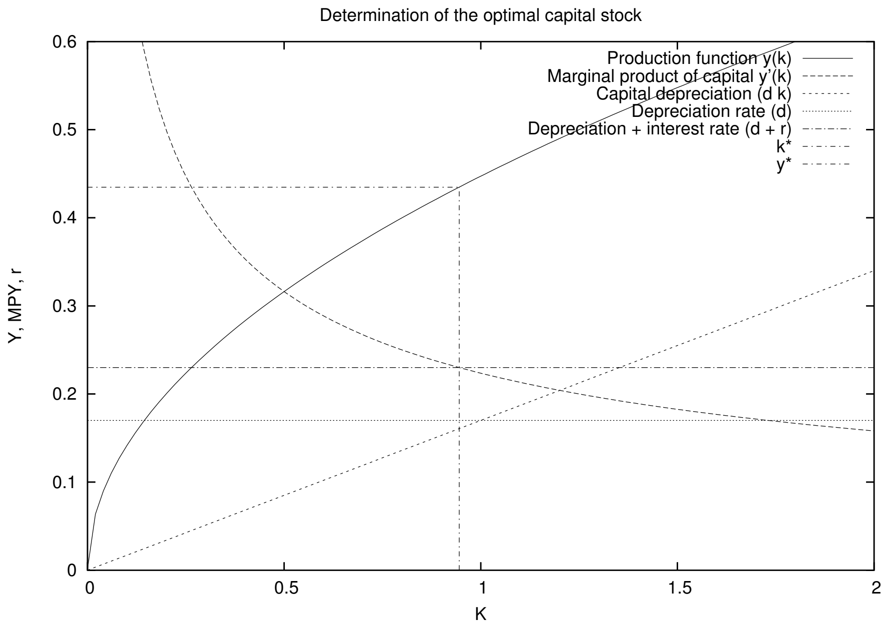

Figure 2: Determination of the equilibrium capital stock

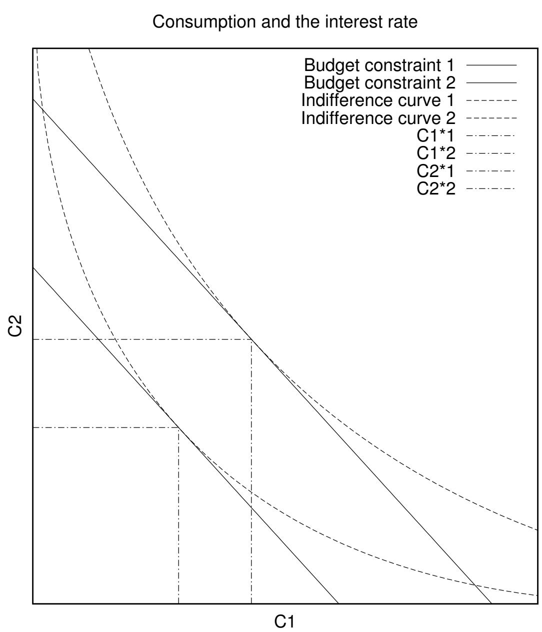

Figure 3: Consumption decrease and savings increase with the interest rate

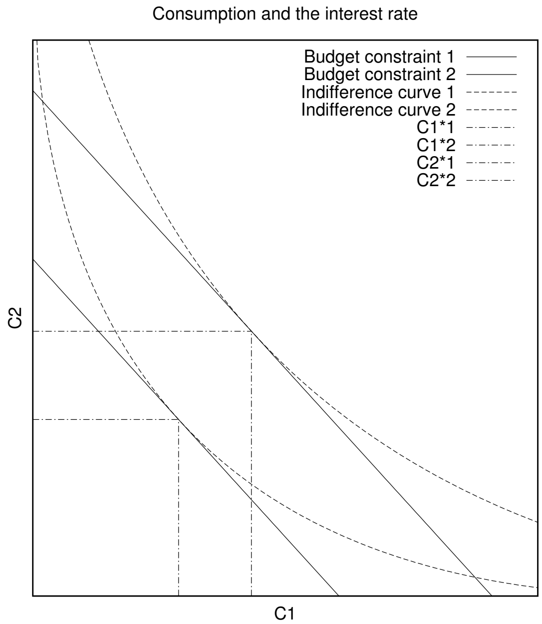

Figure 4: Dependency of consumption and savings on income

Figure 5: Consumption and savings increase with income

Summary of the model

The following equations describe the model.
First two identities.
The goods market equilibrium condition (or national account balance identity):

$$Y = C + I \tag{1}$$

The households' budget constraint:

$$Y = C + S \tag{2}$$

Then the production function and capital restriction, and the firms' decisions that follow from them:
The production function:

$$Y_t = F(K_t, L_t) \tag{3}$$

Capital depreciation and replacement investments:

$$I = \delta K \tag{4}$$

Capital demand:

$$MPK = \frac{\partial F}{\partial K}(K_t, L_t) = \delta + r \tag{5}$$

Labor demand:

$$MPL = \frac{\partial F}{\partial L}(K_t, L_t) = w \tag{6}$$

Next, the households' decisions on consumption and labor supply.

The consumption function:

$$C = c(Y, r) \tag{7}$$

Labor supply:

$$L = \overline{L} \tag{8}$$

And, finally, the monetary side of the economy:

The exchange equation:

$$PY = VM \tag{9}$$

These nine equations determine the nine endogenous variables $Y, C, I, S, K, L, r, w, P$ from the exogenous variables $\delta, \overline{L}, M, V$ and the production and consumption functions.

This system of equations is simultaneous, but not entirely so. First, with fixed labor supply (8), L is determined immediately. Given L, the equations (1), (3), (4), (5), and (7) simultaneously determine Y, C, I, K and r. Given K, labor demand (6) determine the wage w, and given Y and C, savings S are residually dermined by the household budget (2). All real variables are then determined. Finally, given Y, the exchange equation (9) determine the price level P from the money supply M. The model thus features the classical dichotomy, i.e. nominal variables don't influence real ones.

To be continued ...