Efficiency wages and the Solow condition

The basic assumption of the efficiency wage theory is that workers' efficiency increases with their wages. Under such conditions, the firm faces a trade-off between hiring efficient workers at a high wage, or less efficient workers at a lower wage. These notes derive the so called Solow condition for a wage that minimize the cost of effective labor input.

Reasons why efficiency may increase with wages

Several different explanations have been suggested why such a relationship between wages and efficiency might hold.

(a) In a poor country, the market clearing wage may be too low to cover the cost of health care and healthy food, water, and living conditions. At a higher wage, the workers may afford to keep healthier and therefore work more efficiently.

(b) Another explanation is provided by the so called shirking model: At a work place where the employer find it difficult to monitor workers' labor efficiency and output, or where the relationship between labor efficiency and productivity is somewhat random, shirking, i.e. working with low efficiency or doing other things than work during working hours, may be a problem. Higher wages may reduce shirking by making the jobs more attractive to workers, who are then more eager to retain their jobs rather then get caught shirking and perhaps loosing their jobs.

(c) Labor turnover is costly because of hiring and firing costs. Higher wages make it easier to recruit and retain workers by making the firm's jobs more attractive to workers, thus reducing the cost of labor turnover.

(d) A more psychological idea is that wages which are perceived as high will be regarded by the workers as a kind of gift from the employer. They, in exchange, will put more effort into their work. This is sometimes called reciprocity or gift exchange.

The first order condition for cost minimization

Whatever the reason for the relationship between wages and efficiency, the employer then faces a trade-off. He can get a certain amount of work done either by hiring a few workers working efficiently for a high wage, or by hiring more but less efficient workers at a lower wage. The question is which combination of wage and labor efficiency minimizes the cost of a certain amount of effective labor input. The Solow condition gives the first order condition for the cost minimizing combination of wages and worker efficiency.

Let the efficiency of labor e depend on the wage w according to the function $e(w)$ with derivative $e'(w)$. Assume that efficiency is defined in such a way that the product eL of efficiency e and labor L is the relevant labor input in the production function. The product eL is called the amount of effective labor. If labor L is measured in work hours, eL is the number of effective hours; if labor is measured as the number of workers, eL is the number of effective workers.

The production function thus gives output q as a function of capital K and efficient labor eL:

$$q = F(K, e(w)L)$$

Total labor costs are wL, so finding the lowest costs C per efficient worker or efficient work hour leads to the problem of minimizing:

$$C = \frac{wL}{e(w)L} = \frac{w}{e(w)}$$

The first order condition is obtained by setting the derivative of costs with respect to the wage equal to zero:

$$\frac{dC}{dw} = \frac{e(w) - we'(w)}{e^2} = (1 - \frac{w}{e(w)}e'(w))\frac{1}{e(w)} = 0$$

If labor input is to have a positive impact on production, the efficiency e must be positive. Therefore the expression within the parenthesis must equal zero. It follows that:

$$\frac{w}{e(w)}e'(w) = 1$$

This is called the Solow condition. It says that the wage elasticity of worker efficiency is equal to one at the optimal wage level.

Alternatively, the first order condition can be written as:

$$e'(w) = \frac{e(w)}{w}$$

In this form, it says that in a graph with the wage w on the horizontal axis and efficiency e on the vertical axis, the straight line from the origin through the optimal point will be tangent to the efficiency curve $e = e(w)$.

The second order condition

The second order condition says that a wage for which the first order condition is satisfied is a local cost minimum if the second derivative of costs per unit of efficient labor with respect to the wage is positive:

$$\frac{d^2C}{dw^2} = \frac{2e'(w)(we'(w) - e(w)) - we(w)e"(w)}{e^3(w)} > 0$$

According to the first order condition, the parenthesis in the first term of the numerator is zero, so the second order condition reduces to:

$$e"(w) < 0$$

Thus, the second order condition requires that the efficiency function $e = e(w)$ is concave downward at the optimal point, i.e. the marginal efficiency gain from wages is decreasing.

The first order conditions for profit maximization

The fact that the Solow condition follows from cost minimization, and so does not require profit maximization, means that it can be expected to hold not only for firms, but equally well for other employers such as government agencies and household organizations.

Obviously profit maximization entails cost minimization. Nevertheless it may be instructive to see how the Solow condition is derived from the first order conditions for profit maximization.

Suppose that a firm produces its output q according to the production function

$$q = F(K, e(w)L)$$

and sells it on a product market at a price p. If the product market is competitive, the price is constant, but in order to allow for the possibility that the firm wields some market power on its product market, assume that it faces the inverse product demand curve $p = p(q)$. It buys or rents its capital K on a competitive market for physical capital at a rental cost r, and can hire any amount of labor L, which provides effective labor services depending on the wage w according to the efficiency function $e = e(w)$.

The firm chooses its amount of capital K and labor L, as well as the wage w that it pays its workers, to maximize its profits:

$$\pi = p(q)q - rK - wL$$

The first order conditions with respect to K, L, and w set the derivatives of profits with respect to these three variables equal to zero:

$$\frac{\partial \pi}{\partial K} = (p(q) + q\frac{dp}{dq})\frac{\partial F}{\partial K}(K, e(w)L) - r = 0$$

$$\frac{\partial \pi}{\partial L} = (p(q) + q\frac{dp}{dq})e(w)\frac{\partial F}{\partial L}(K, e(w)L) - w = 0$$

$$\frac{\partial \pi}{\partial w} = (p(q) + q\frac{dp}{dq})Le'(w)\frac{\partial F}{\partial L}(K, e(w)L) - L = 0$$

The first of these conditions simply states the usual condition for an optimal capital stock, that the marginal revenue product of capital is equal to the rental cost of capital:

$$MRP_K = (p + q\frac{dp}{dq})\frac{\partial F}{\partial K} = r$$

The second condition requires that the marginal revenue product of effective labor input be equal to the cost per unit of effective labor:

$$MRP_L = (p + q\frac{dp}{dq})\frac{\partial F}{\partial L} = \frac{w}{e(w)}$$

Substituting the second condition into the third, results in the condition that the wage w is chosen so that these costs of effective labor are minimized:

$$\frac{w}{e(w)}e'(w) = 1$$

This is, again, the Solow condition.

Note that the Solow condition is independent of the firm's technology, i.e. the production function, and of the conditions on the product market, i.e. the product demand function.