## A firm's labor choice under imperfect competition

A firm operating under imperfect competition on its labor- and/or product markets has a somewhat more complex employment decision problem compared to the competitive firm. These notes develop the theory of firm employment under imperfectly competitive labor- and product markets, and compare the results with the case of competitive markets.

## Labor demand under perfect competition

Consider first the case where markets are competitive. Suppose, for simplicity, that the firm produces its output using labor as it only input according to the production function

$$
q=q(E)
$$

so the marginal product of labor is

$$
M P_{E}=\frac{d q}{d E}
$$

The competitive firm cannot influence its product price $p$ or the wage $w$, so it treats these prices as given constants. Profits $\pi$ are revenue $R=p q$ minus costs $C=w E$, thus

$$
\pi=R-C=p q(E)-w E
$$

The first order condition for optimal employment is that the derivative of profits with respect to employment is zero:

$$
\frac{d \pi}{d E}=p \frac{d q}{d E}-w=p M P_{E}-w=0
$$

Thus, a firm selling its product on a competitive product market and hiring labor on a competitive labor market has an inverse labor supply function equal to its value marginal product of labor $\left(V M P_{E}\right)$.

$$
w=V M P_{E}=p M P_{E}=p \frac{d q}{d E}(E)
$$

## Monopsonist labor market and monopolist product market

If the firm operates on non-competitive product- and labor markets, this result is modified. Then, the price $p$ that the firm recieves for its product and the wage $w$ it pays for its labor are no longer constants given by the markets. Assume instead that the firm has a monopoly on the product market, i.e. it's the only seller on the market. Then the product price $p$ depends negatively on the quatity $q$ the firm sells on the product market, according to the inverse product demand function

$$
p=p(q)
$$

Assume also that the firm is a monopsonist on the labor market, i.e. it's the only employer on the market. Then the wage $w$ increases with the amount of labor $E$ that the firm employs according to the inverse labor supply function

$$
w=w(E)
$$

Revenue $R$ now depends on employment according to

$$
R=p q=p(q) q=p(q(e)) q(E)
$$

Marginal revenue is the derivative of revenue with respect to production:

$$
M R=\frac{d R}{d q}=\frac{d p}{d q} q+p
$$

Costs $C$ vary with employment according to

$$
C=w E=w(E) E
$$

The marginal labor cost $M C_{E}$ is the derivative of costs with respect to employment:

$$
M C_{E}=\frac{d C}{d E}=\frac{d w}{d E} E+w
$$

The firm's profits $\pi$ are revenue minus costs

$$
\pi=R-C=p q-w E=p(q(e)) q(E)-w(E) E
$$

Just like for the competitive firm, the first order condition for optimal employment states that the derivative of profits with respect to employment is zero:

$$
\frac{d \pi}{d E}=\frac{d R}{d E}-\frac{d C}{d E}=\frac{d R}{d q} \frac{d q}{d E}-\frac{d C}{d E}=M R M P_{E}-M C_{E}=0
$$

The condition for optimal employment thus says that the firm's employment should be chosen so that the marginal revenue times the marginal product of labor is equal to the marginal cost of labor.

$$
M R M P_{E}=M C_{E}
$$

The product of marginal revenue and the marginal product of labor is called the marginal revenue product of labor, $M R P_{E}$ :

$$
M R P_{E}=M R M P_{E}
$$

Thus the first order condition for optimal employment states that the marginal revenue product of labor equals the marginal labor cost:

$$
M R P_{E}=M C_{E}
$$

## One imperfectly competitive market, the other competitive

In the case where the labor market is competitive, the wage is independent of employment, so

$$
\frac{d w}{d E}=0
$$

and the marginal labor cost is equal to the wage $w$.

$$
M C_{E}=\frac{d C}{d E}=\frac{d w}{d E} E+w=w
$$

Thus, in this case, the first order condition for optimal employment states that the marginal revenue product is equal to the wage:

$$
M R M P_{E}=w
$$

In the case where the product market is competitive, the product price is independent of the quantity sold, so

$$
\frac{d p}{d q}=0
$$

and the marginal revenue is equal to the price $p$.

$$
M R=\frac{d R}{d q}=\frac{d p}{d q} q+p=p
$$

Thus, in this case, the first order condition for optimal employment states that the value marginal product is equal to the marginal labor cost:

$$
p M P_{E}=M C_{E}
$$

## Summary

The first order conditions for a firm's optimal employment decision are summarized in the following table.

| Product market | Labor market |  |
| :--- | :--- | :--- |
| Competitive | Competitive | Monopsony |
|  | $p M P_{E}=w$ | $p M P_{E}=M C_{E}$ |
|  | $M R M P_{E}=w$ | $M R M P_{E}=M C_{E}$ |

