Exercises in Labor economics (NAA203)

Some useful exercises adapted from old editions of *Labor economics* by George J. Borjas

6. ed. Ex. 4-9

A firm produces an output good using labor as its only input, according to the production function

$$q = 2E$$

where q is the number of units produced, and E is the number of hours worked.

It has a monopoly on its product and, being the only employer in the area, it's a monopsonist on its labor market. On the product market, the firm faces the inverse demand curve

$$p = 30 - 0.4q$$

where p is the price per unit. It hires labor according to the inverse labor supply curve

$$w = 0.9E + 5$$

where w is the hourly wage.

(a) What is the optimal employment level E?

(b) What wage w will the firm pay?

(c) How much output q will the firm produce?

(d) What price p will the firm charge for its product?

(e) What will be the firm's profits?

Solution

(a) The firm is assumed to maximize profits

$$\pi = pq - wE$$

Substituting the inverse product demand for p, the inverse labor supply for w, and the production function for q yields

$$\pi = (30 - 0.4q)q - wE = (30 - 0.4(2E))(2E) - (0.9E + 5)E$$

which simplifies to

$$\pi = 55E - 2.5E^2$$

At profit maximum, the derivative of profits with respect to employment is zero, so

$$\frac{d\pi}{dE} = 55 - 5E = 0$$

Solving for employment yields

$$E = 11$$

(a) Alternatively use the first order condition that the marginal revenue times the marginal product of labor is equal to the marginal cost of labor.

$$\mathrm{MR}\ \mathrm{MP}_E = \mathrm{MC}_E$$

Marginal revenue is the derivative of revenue with respect to quantity (q):

$$\mathrm{MR} = \frac{d}{dq}(pq) = \frac{d}{dq}(30 - 0.4q)q = 30 - 0.8q = 30 - 0.8(2E) = 30 - 1.6E$$

The marginal product of labor is the derivative of production with respect to labor (E):

$$\mathrm{MP}_E = \frac{dq}{dE} = \frac{d}{dE}(2E) = 2$$

The marginal cost of labor is the derivative of costs with respect to labor (E):

$$\mathrm{MC}_E = \frac{d}{dE}(wE) = \frac{d}{dE}(0.9E + 5)E = 1.8E + 5$$

Substituting these marginal effects into the optimum conditions yields

$$(30 - 1,6E)2 = 1.8E + 5$$

Solving for employment yields

$$E = 11$$

(b) The wage w is calculated by substituting employment $E = 11$ into the inverse labor supply function.

$$w = 0.9E + 5 = 0.9 \cdot 11 + 5 = 14.9$$

(c) Substituting employment $E = 11$ into the production function yields production q.

$$q = 2E = 2 \cdot 11 = 22$$

(d) The product price p is obtained by substituting production $q = 22$ into the inverse product demand function.

$$p = 30 - 0.4q = 30 - 0.4 \cdot 22 = 21.2$$

(e) Profits π are calculated as

$$\pi = pq - wE = 21.2 \cdot 22 - 14.9 \cdot 11 = 302.5$$

6. ed. Ex. 5-12

A firm produces an output good using labor as input. The firm has contracted to supply $q = 2\,000$ units of output for the price $p = 700$ per unit. To produce this amount, the firm must employ $E = 20$ workers.

The firm must choose how much to invest in a safe working environment. The firm can choose any level of safety, S, from 0 to 100. The cost of safety is $C(S) = 50S^2$.

Given the firm's choice of safety, the wage payed to workers is

$$w = 60\,000 - 300S$$

(a) What level of safety (S) will the firm choose?

(b) What is the cost $C(S)$ of this optimal safety level?

(c) What wage (w) will each worker earn?

(d) What are the firm's profits?

Solution

(a) The firm's costs C of hiring $E = 20$ workers are wage costs and the cost of safety.

$$C = wE + C(S)$$

Substituting the wage function, the safety cost and the employment level yields:

$$C = wE + C(S) = 20(60\,000 - 300S) + 50S^2 = 1\,200\,000 - 6\,000S + 50S^2$$

The firm would like to minimize this cost. At cost minimum, the derivative of costs with respect to safety is zero.

$$\frac{dC}{dS} = -6\,000 + 100S = 0$$

With the solution for optimal safety

$$S = 60$$

(b) The cost of safety is

$$C(S) = 50S^2 = 50(60)^2 = 180\,000$$

(c) The wage at safety level $S = 60$ is

$$w = 60\,000 - 300S = 60\,000 - 300 \cdot 60 = 42\,000$$

(d) Profits are

$$\pi = pq - wE - C(S) = 700 \cdot 2\,000 - 42\,000 \cdot 20 - 180\,000 = 380\,000$$

6. ed. Ex. 11-13

A firm produces an output good using labor as its only input. Total output is proportional to the number of workers employed. The relationship between a worker's dayly wage, w, and his dayly output, q, is

$$q = 0.1w^2 - 0.0005w^3$$

(a) What is the optimal efficiency wage (w) for the firm to pay?

(b) How much output (q) will each worker produce?

(c) Suppose the firm sells its output at the constant price $p = 0.80$ per unit. How much dayly profit does the firm earn on each worker's output?

Solution

(a) In order to maximize profits, the firm would choose a wage to minimize its cost per unit of output is w/q. This is equivalent to maximizing the inverse of unit cost, q/w, which in this case leads to simpler mathematical expressions. An expressioin for the inverse unit cost is

$$\frac{q}{w} = \frac{0.1w^2 - 0.0005w^3}{w} = 0.1w - 0.0005w^2$$

The first order condition for a maximum is

$$\frac{d}{dw}\left(\frac{q}{w}\right) = 0.1 - 0.001w = 0$$

Solving for w yields

$$w = 100$$

(a) Alternatively use the Solow condition:

$$\frac{w}{q}\frac{dq}{dw} = 1$$

or equivalently

$$\frac{dq}{dw} = \frac{q}{w}$$

i.e. the marginal product of the wage is equal to the inverse unit cost. The marginal product of the wage is

$$\frac{dq}{dw} = 0.2w - 0.0015w^2$$

and the inverse unit cost is

$$\frac{q}{w} = 0.1w - 0.0005w^2$$

Substituting these expressions into the Solow condition yields

$$0.2w - 0.0015w^2 = 0.1w - 0.0005w^2$$

This quadratic equation has two solutions. One is $w = 0$, which is optimal only if it's impossible to earn positive profits. The other solution is obtained by dividing both sides by w.

$$0.2 - 0.0015w = 0.1 - 0.0005w$$

Solving for w yields

$$w = 100$$

(b) Production per worker is

$$q = 0.1w^2 - 0.0005w^3 = 0,1(100)^2 - 0.0005(100)^3 = 500$$

(c) Profits per worker are

$$\pi = pq - w = 0.8 \cdot 500 - 100 = 300$$

6. ed. Ex. 12-8

The population of a country can be divided into the employed, the unemployed, and the persons who are out of the labor force (OLF).

In any given year, the transition probabilities among the various categories are given by the following table.

		Moving into		
		Employed	Unemployed	OLF
Moving from:	Employed	0.94	0.02	0.04
	Unemployed	0.20	0.65	0.15
	OLF	0.05	0.03	0.92

The transition probabilities are interpreted as follows. In any given year, 2% of the employed become unemployed, 20% of the unemployed find jobs, and so on.

(a) What is the steady state employment rate?

(b) What is the steady state unemployment rate?

(c) What is the steady state labor force participation rate?

Solution

Denote the number of people in the population by P, the number of employed by E, and the number of unemployed by U. Let $e = E/P$ be the share of the population which are employed, and $u' = U/P$ the share of the population which are unemployed. Note that u' is not the unemployment rate. The unemployment rate u is the share of the labor force that are unemployed:

$$u = \frac{U}{E+U} = \frac{u'P}{eP + u'P} = \frac{u'}{e+u'}$$

The share of the population that is out of the labor force is $1 - e - u'$.

The flow of workers out of employment into unemployemt and out of the labor force (OLF) is

$$(0.02 + 0.04)e = 0.06e$$

The flow into employment from unemployment and OLF is

$$0.20u' + 0.05(1 - e - u')$$

Steady state requires that inflow into employment equals outflow from employment.

$$0.20u' + 0.05(1 - e - u') = 0.06e$$

Likewise, the flow out of unemployment into employemt and OLF is

$$(0.20 + 0.15)u' = 0.35u'$$

The flow into unemployment from employment and from OLF is

$$0.02e + 0.03(1 - e - u')$$

Again, steady state requires that inflow equals outflow.

$$0.02e + 0.03(1 - e - u') = 0.35u'$$

The steady state values of the two variables e and u' are determined by the two steady state equations

$$0.20u' + 0.05(1 - e - u') = 0.06e$$

$$0.02e + 0.03(1 - e - u') = 0.35u'$$

Algebraic rearanging brings them into the so called normal form

$$0.11e - 0.15u' = 0.05$$

$$0.01e + 0.38u' = 0.03$$

Multiplying the second equation by 11 and subracting the first yields

$$4.33u' = 0.28$$

Thus

$$u' = \frac{0.28}{4.33} = 0.064665$$

Solving the second equation above for e yields

$$e = 3 - 38u'$$

Substituting the value for u' gives

$$e = 3 - 38\frac{0.28}{4.33} = 0.54273$$

So the solution to the equation system for steady state is $e = 0.54273, u' = 0.064665$

(a) The steady state employment rate is

$$e = 0.54273 = 54\%$$

(b) The steady state unemployment rate is

$$u = \frac{u'}{e + u'} = \frac{0.064665}{0.54273 + 0.064665} = 0.10646 = 10.6\%$$

(c) The steady state labor force participation rate is

$$\frac{E + U}{P} = e + u' = 0.54273 + 0.064665 = 0.60739 = 60.7\%$$